Effects of Mistuning on the Forced Response of Bladed Discs with Friction Dampers

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ABSTRACT

A method recently developed by the authors allows efficient calculation of the periodic forced response to be performed for bladed discs with arbitrary nonlinearities, including friction contacts and gaps. The method can be used with large finite element models and includes friction contact models, allowing for variable normal stresses at the fiction contact interfaces, such as underplatform dampers, blade-root joints and others. A multiharmonic representation of the forced response enables analysis to be undertaken with the required accuracy and under arbitrary distributed and multiharmonic excitation.

In this paper, the effects of friction dampers on the multiharmonic forced response of large-scale finite element models of practical mistuned bladed discs are studied for the first time. Maximum forced response levels of mistuned systems are explored and compared with those for tuned assemblies. The effects of parameter values for the friction dampers on the amplification factors and distribution of the maximum amplitudes over the bladed disc assembly are investigated. Conditions are found when the amplification factors can be <u>*decreased*</u> *as a result of mistuning, in direct contrast to the conventional results obtained using linear models.*

1.0 INTRODUCTION

It is widely known that even small degrees of blade mistuning can result in a significant increase of forced response levels of bladed discs compared with those of their tuned counterparts under the same excitation conditions. Mistuning effects on forced response, when linear models of bladed discs are applied, have been investigated during recent years using large-scale finite element models. However, linearised models cannot generally be applied to a large range of practical bladed discs, including those with friction dampers fitted. These models, in many cases, cannot capture even qualitatively the influence of friction and varying stiffness of the blade-damper interactions on forced response, and a loss of accuracy of the calculations is inevitable. Analysis of mistuned bladed discs with friction dampers, for which vibrations are essentially nonlinear, has not been performed hitherto with large-scale models to the extent necessary to understand fully the mistuning effects on forced response of the bladed discs with friction damping.

Practical bladed discs cannot be perfectly tuned, i.e. cannot be cyclically symmetric, due to the inevitable imperfections occurring during manufacturing and operation. It is known that violation of the bladed disc cyclic symmetry – mistuning – can result in an increase of the forced response levels of several times even when such perturbations are small and satisfy manufacture tolerance limits. Moreover, bladed disc mistuning can cause large scatters in amplitudes of different blades of a mistuned assembly which contrasts to the case of its tuned counterpart when blade amplitudes are identical.

Bladed disc mistuning is caused not only by a scatter in the characteristics of the blades but also by a

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scatter of parameters of the underplatform dampers, blade-disc joints and/or contact interfaces. To date, mistuning investigations have been restricted mostly to the analysis of the effects of blade frequency mistuning and by the assumption that vibration of a structure analysed is linear. Even with these restrictions, the problem is computationally complex and although mistuning problem have attracted a major interest of industries and scientists for more than 40 years, most of the studies have been made using simplified models of bladed discs. Recently, for the analysis of linear vibrations of mistuned bladed discs several techniques have been developed which are intended for use with models having large numbers of degrees of freedom (see papers [1]-[4]).

However, in practical bladed disc assemblies, nonlinear contact interaction forces commonly occur – at blade-disc joints, in friction damper devices such as underplatform dampers, and at the contact surfaces of adjacent shrouds in shrouded bladed discs, etc. These contact interaction forces can make the behaviour of the structure strongly nonlinear, so that it exhibits phenomena that cannot be described and predicted by linearised models.

A pioneering method for calculation of the forced response for essentially nonlinear vibrations of realistic mistuned bladed discs has been presented in reference [5] and [6]. This method allows realistic large-scale finite element models to be used in nonlinear forced response analysis of mistuned bladed discs. Another approach to analysis of nonlinear structures including mistuned ones, has been proposed in [7]. Although there are examples of calculations of nonlinear forced response for mistuned bladed discs in these papers, but, with such very restricted studies of the nonlinear mistuned forced response available in a literature, there is a need to perform further analyses of mistuning effects which are specific for essentially nonlinear bladed discs.

In this paper a forced response analysis is performed for mistuned bladed discs with underplatform dampers (UPDs) in order to understand effects of UPDs on the mistuned forced response. In particular, the goals of the study include the following:

- to understand how nonlinear contact interfaces affect forced response characteristics of mistuned assemblies, such as amplitude levels, amplification factors, amplitudes scatters;
- to determine whether the trends found as results of investigations for linear mistuned assemblies are different from the nonlinear ones;
- to assess the effects of scatter in the friction interface parameters in addition to conventional blade frequency mistuning.

The numerical investigation is performed using the method reported in [6] and realising its program code included into nonlinear forced response analysis suite, FORSE, developed at Imperial College London.

2.0 METHODOLOGY OF THE NONLINEAR MISTUNING ANALYSIS

2.1 Method for analysis of multiharmonic forced response for mistuned bladed discs

The method used in the calculations accounts for the major kinds of mistuning in bladed discs, such as: (i) scatter in the dynamic characteristics of individual blades, disc sectors and damping devices, and (ii) scatter in the characteristics of contact interfaces of an assembled structure. For the purpose of developing a highly efficient calculation method, all mistuning sources are separated in two categories: (i) nonlinear mistuning, which results from scatter of parameters of the nonlinear friction contact interfaces; and (ii) linear mistuning, which can be described by modification of the linear components of the assembly (e.g. blade frequency mistuning, blade damping mistuning, etc.). The equation of motion of a mistuned bladed disc can be written in the form:

$$\boldsymbol{Kq} + \boldsymbol{C\dot{q}} + \boldsymbol{M\ddot{q}} + \boldsymbol{f}(\boldsymbol{q}) = \boldsymbol{p}(t) \tag{1}$$

where $\boldsymbol{q}(t)$ is a vector of displacements for all the degrees of freedom (DOFs) in the mistuned assembly. $\boldsymbol{K}=\boldsymbol{K}_0+\delta\boldsymbol{K}$, $\boldsymbol{C}=\boldsymbol{C}_0+\delta\boldsymbol{C}$ and $\boldsymbol{M}=\boldsymbol{M}_0+\delta\boldsymbol{M}$ are, respectively, the stiffness, viscous damping and mass matrices of the mistuned bladed disc. Each of these matrices is expressed as the sum of matrix of a perfectly tuned bladed disc, $\boldsymbol{K}_0$, $\boldsymbol{C}_0$ or $\boldsymbol{M}_0$, and a mistuning modification matrix, $\delta\boldsymbol{K}$, $\delta\boldsymbol{C}$ or $\delta\boldsymbol{M}$, caused by mistuning of a linear nature. $\boldsymbol{f}(\boldsymbol{q})$ is a vector of non-linear forces occurring at friction contact interfaces, which are nonlinearly dependent on displacements and can differ for different bladed disc sectors making the bladed disc to be classified as "nonlinear mistuned". $\boldsymbol{p}(t)$ is a vector of periodic external excitation forces, i.e. $\boldsymbol{p}(t)=\boldsymbol{p}(t+T)$, with a period, $T=2\pi/\omega$, determined by the rotor rotation speed, ω.

Linear mistuning is modelled by special linear mistuning elements which can describe any desired type of linear modification matrices, $\delta\boldsymbol{K}$, $\delta\boldsymbol{C}$ or $\delta\boldsymbol{M}$. Examples of using the linear mistuning elements for modelling blade frequency mistuning are given in [2]. Exact and highly numerically-efficient friction contact elements developed in [8] are used for modelling of friction contact interfaces. The capability developed to describe parameters of friction contact for every contact patch and every contact node individually which allows the analysis of any kind of nonlinear mistuning.

To determine the steady-state periodic nonlinear forced response, a multiharmonic representation of displacements and excitation forces is used, which can contain as many and such harmonic components as are necessary to approximate the sought solution with the required accuracy, i.e.

$$\boldsymbol{q}(t)=\sum_{j=1}^{n}\boldsymbol{Q}_j^{(c)}\cos m_j\omega t+\boldsymbol{Q}_j^{(s)}\sin m_j\omega t\,;\quad \boldsymbol{p}(t)=\sum_{j=1}^{n}\boldsymbol{P}_j^{(c)}\cos m_j\omega t+\boldsymbol{P}_j^{(s)}\sin m_j\omega t \tag{2}$$

where $\boldsymbol{Q}_j^{(c)}$, $\boldsymbol{Q}_j^{(s)}$ and $\boldsymbol{P}_j^{(c)}$, $\boldsymbol{P}_j^{(s)}$ $(j=1...n)$ are vectors of harmonic coefficients for $\boldsymbol{q}(t)$ and $\boldsymbol{p}(t)$ respectively, and m_j $(j=1\ldots n)$ are numbers of harmonics that are chosen to be retained in the multiharmonic expansion. Substitution of Eq.(2) in Eq.(1) gives a nonlinear algebraic equation with respect to the harmonic coefficients of the multiharmonic expansion. Formulation of the multiharmonic equation of motion in a form using complex arithmetic as developed in paper [9], provides an expression in the following form:

$$\text{diag}[\boldsymbol{K}+im_1\omega\boldsymbol{C}-(m_1\omega)^2\boldsymbol{M},...,\boldsymbol{K}+im_n\omega\boldsymbol{C}-(m_n\omega)^2\boldsymbol{M}]\boldsymbol{Q}+\boldsymbol{F}(\boldsymbol{Q})=\boldsymbol{P} \tag{3}$$

where $\boldsymbol{P}=\left\{\boldsymbol{P}_1^{(c)}-i\boldsymbol{P}_1^{(s)},\ldots,\boldsymbol{P}_n^{(c)}-i\boldsymbol{P}_n^{(s)}\right\}^T$ is a vector of harmonic components of the excitation forces and $\boldsymbol{Q}=\left\{\boldsymbol{Q}_1^{(c)}-i\boldsymbol{Q}_1^{(s)},\ldots,\boldsymbol{Q}_n^{(c)}-i\boldsymbol{Q}_n^{(s)}\right\}^T$ is the unknown vector of harmonic coefficients of displacements. One can see that sizes of the matrices and vectors in Eq.(3) are proportional to the number of harmonics kept in the solution.

Realistic finite element (FE) models used in practical applications usually contain ~10^5..10^6 DOFs and this can make mistuning analysis very time-consuming, even for the case of linear systems. For the case of nonlinear forced response analysis, when iterative solution of nonlinear Eq.(3) is unavoidable and use of many harmonics in the multiharmonic expansion increases the size of the matrices by several times, a direct solution of the nonlinear equations is not feasible. Because of this, a two-stage method of exact condensation of the nonlinear equation has been developed. A scheme for this calculation is shown in Figure 1.

As a first step a model of a tuned bladed disc is constructed from its sector FE model keeping the DOFs at which linear and nonlinear mistuning elements are to be applied and the all other DOFs are condensed without loss of completeness and accuracy of modelling. Then linear mistuning elements are applied to the whole tuned bladed disc to describe modifications of the mass, $\delta\boldsymbol{M}$, stiffness, $\delta\boldsymbol{K}$, and damping, $\delta\boldsymbol{C}$, characteristics of the assembly caused by linear mistuning (including mistuning due to scatter of blade natural frequencies and modal damping factors). At second condensation step all linear DOFs are excluded and the model matrices are again exactly condensed. At the end of this step only nonlinear DOFs are explicitly present. Nonlinear modification elements are applied to these nodes and the nonlinear equations are solved with respect to only the nonlinear DOFs. The number of nonlinear DOFs is usually small and hence a high speed of calculation is achieved for the forced response. Detailed description of the method of calculation can be found in [6].

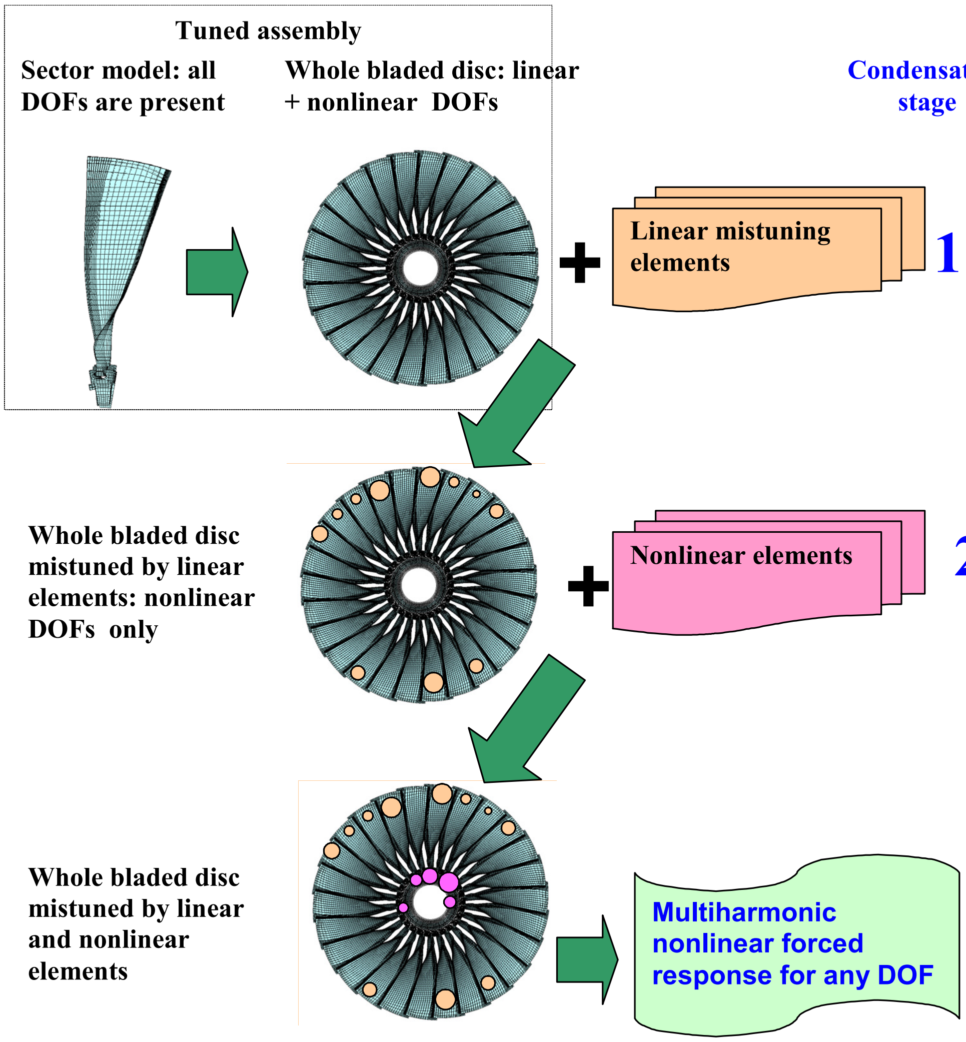


Figure 1: Scheme of the developed multistage reduction method of the number of DOFs kept in resolving equations while preserving model accuracy and completeness

2.2 Characteristics of the mistuned forced response analysed

In reference [9] it was shown that, when a tuned bladed disc is excited by forces of travelling wave type, blade amplitudes are identical even in cases of strongly nonlinear nature of system and arbitrary distribution of the excitation forces over the bladed disc. For a mistuned bladed disc, in contrast to a tuned structure, the forced response characteristics are different for each of its blades, as illustrated in Figure 2. In order to provide overall characterization of the forced response for mistuned structures, the following major characteristics, schematically shown in Figure 2, are used:

(i) an envelope of the maximum forced response searched for each excitation frequency over all blades, i.e.

$$a_{env}(\omega) = \max_{j=1\ldots N_b}\left(a_j(\omega)\right) \tag{4}$$

(ii) the maximum blade response levels found for each blade over the frequency range analysed, i.e.

$$a_j^{\max} = \max_{\omega\in[\omega^-,\omega^+]}\left(a_j(\omega)\right) \tag{5}$$

(iii) the maximum response level found over all blades of a bladed disc and over the whole frequency range analysed, i.e.

$$a_\Sigma^{\max} = \max_{j=1\ldots N_b}\left(\max_{\omega\in[\omega^-,\omega^+]}\left(a_j(\omega)\right)\right) \tag{6}$$

where ω^- and ω^+ are the lower and higher bounds of the frequency range analysed, respectively, and N_B is the number of blades in the bladed disc. One can see that these characteristics are similar to those used in the majority of linear mistuning studies. However, since the analysed vibration are periodic and not necessary harmonic, the maximum blade response which is introduced here is determined over period of vibration, i.e. $a_j = \max_{t\in[0,T]}\left(\sqrt{x_j^2(t)+y_j^2(t)+z_j^2(t)}\right)$ instead of the vibration amplitudes typically used in linear mistuning studies where responses are monoharmonic. Moreover, in order to compare forced response levels of a mistuned assembly with that of its tuned counterpart normalised values of these characteristics are used, where the normalisation coefficient is the maximum forced response of a tuned assembly, i.e. $a_{tuned}^{\max} = \max_{\omega\in[\omega^-,\omega^+]}\left(a_{tuned}(\omega)\right)$.

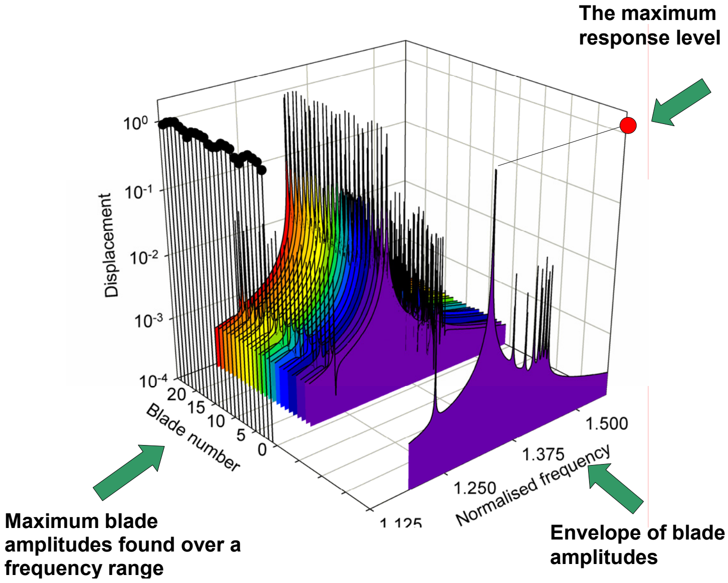


Figure 2: Forced response characteristics used in analysis of mistuned bladed discs

3.0 NUMERICAL INVESTIGATIONS

3.1 A blisk with underplatform dampers

The first test case uses a blisk which analysed was manufactured for a test rig built at Imperial College London in the framework of the EU project “Aeroelastic Design of Turbine Blades II” (ADTurbII) (see [12]). A finite element model of this blisk is shown in Figure 3. The model of one sector of the blisk contains 7185 nodes and the complete blisk comprises 24 blades (Figure 3a). The total number of degrees of freedom (DOFs) in the model of the mistuned bladed (Figure 3b) is (7185×3)×24=517,320. The cottage-roof underplatform dampers (Figure 3d) were fitted into special slots and contacted with blades at nodes marked by letters 'B' and 'C' in Figure 3c. Natural frequencies of the tuned bladed disc without underplatform dampers are plotted in Figure 3e for all nodal diameters (ND) from 0 to 12. Excitation forces are applied to each blade of the blisk at a node located in the middle of the shroud (this node is marked by letter 'A' in Figure 3a). A travelling wave type of excitation of 19th engine order (19EO) is considered. The backward travelling 19EO excitation considered here excites modes of the tuned assembly with 24-19=5 nodal diameters (ND) which travel forward and the frequency range considered is shown in Figure 3e by a red line. Inherent damping was assumed to be equal for all modes involved in the blisk forced response analysis. Since there is no root or other contact interfaces in the blisk and friction damping due to UPDs is included into the model separately the modal damping is solely due to damping in material of the blisk and the damping loss factor is low: $\eta = 7.5\text{e-}05$. Blade frequency mistuning considered is less than $\pm 1\%$.

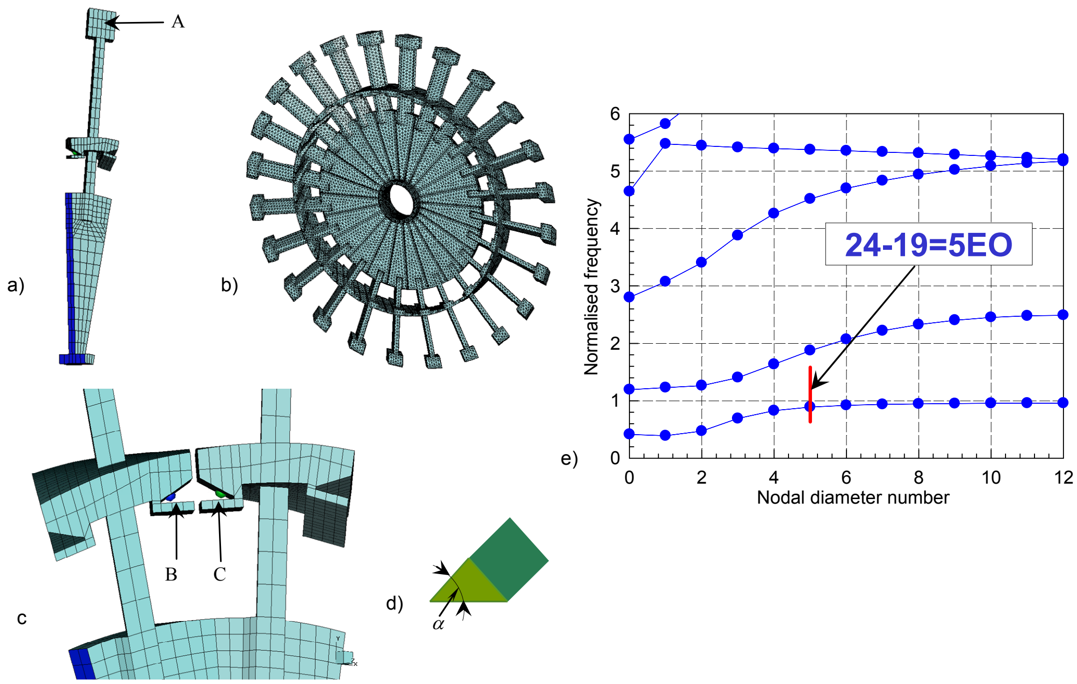


Figure 3: A model of the ADTurbII test piece: a) a blisk sector; b) a whole blisk; c) underplatform damper contact nodes, d) an underplatform damper

The envelopes of mistuned forced response are compared in Figure 4 together with those for their tuned counterparts for two extreme cases: (i) for the blisk without UPDs and (ii) for the blisk with fully stuck UPDs. In both these cases there is no friction damping and the system is linear. One can see that for the mistuned blisk without UPDs, a multitude of secondary resonance peaks are present in addition to the major resonance peaks corresponding to modes of 5ND family – the only one that can be excited for the case of the tuned blisk.

Envelopes of the forced response for the blade-frequency-mistuned blisk with tuned characteristics of underplatform dampers are shown in Figure 5 for different levels of the limiting friction force, μN_0 (where μ is the friction coefficient and N_0 is the normal load at contact surfaces of the UPDs). One can see that the UPDs suppress the secondary resonance peaks for all values of the limiting friction force UPDs except at its higher values, 200% and 1000%. These high limiting friction force values ensure that the UPDs start to slip and, hence, produce friction damping for the high response levels which can be excited only in close proximity to the response peaks. For all other excitation frequency ranges, the UPDs are stuck at these values of the limiting friction force.

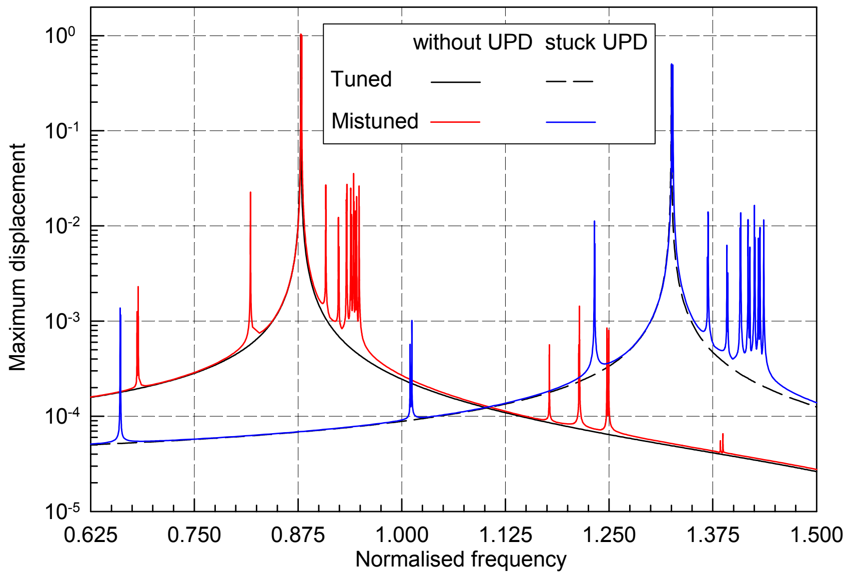


Figure 4: Envelopes of the forced response of the mistuned blisk with stuck UPDs and without UPDs compared with the response of tuned counterparts

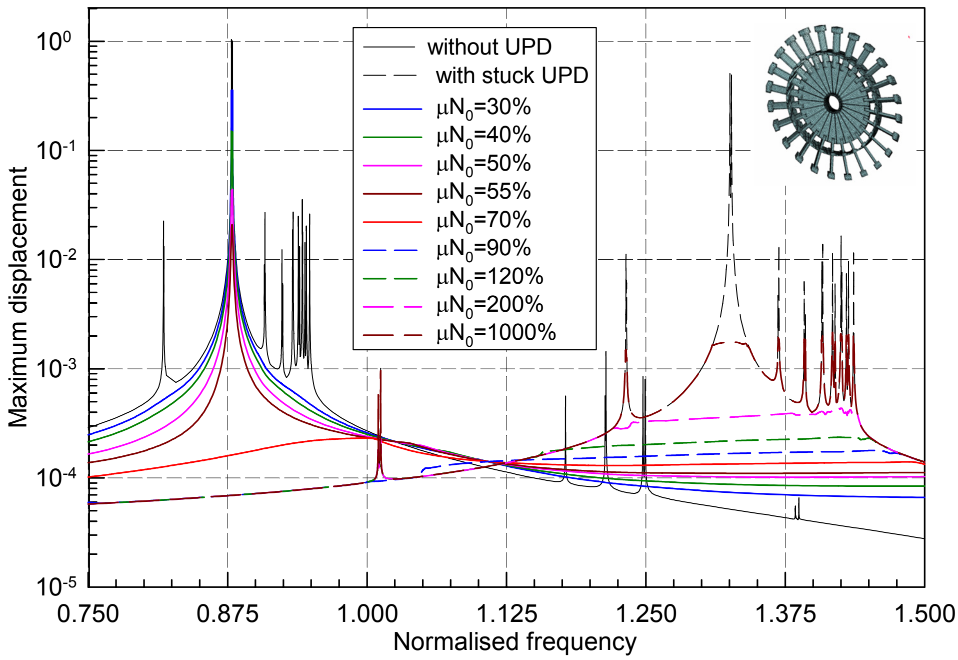


Figure 5: Forced response of the mistuned blisk with underplatform dampers calculated for different levels of the limiting friction force

Envelopes for the forced response levels calculated for mistuned assemblies are compared with forced responses of tuned counterparts in Figure 6. These forced responses are plotted in Figure 6a for low values of the limiting friction force, μN_0, (from 30% to 55%) and in Figure 6b they are shown for higher values of μN_0 (from 70% to 1000%). An important new effect is observed here for low values of the limiting friction force: namely, the possibility for a mistuned bladed disc to have a forced response level which is significantly lower than that for a tuned assembly with the same parameters. This contrasts to the effect of mistuning for linear structures with randomly chosen mistuning patterns has been reported many times in the literature in the past. This response reduction results directly from the action of the underplatform dampers and is not a feature of a special choice of mistuning pattern. This reduction is likely to be related to the different amount of damping produced by the UPDs for mistuned and tuned structures. Energy dissipated by each damper is dependent on a relative displacement between adjacent blades and for a mistuned structure there is usually a large scatter in relative motion between blades. Due to the nonlinear dependency of the friction forces on relative displacements, such scatter in the friction dampers can produce higher total energy dissipation for some damper parameters in a mistuned bladed disc than for a tuned one.

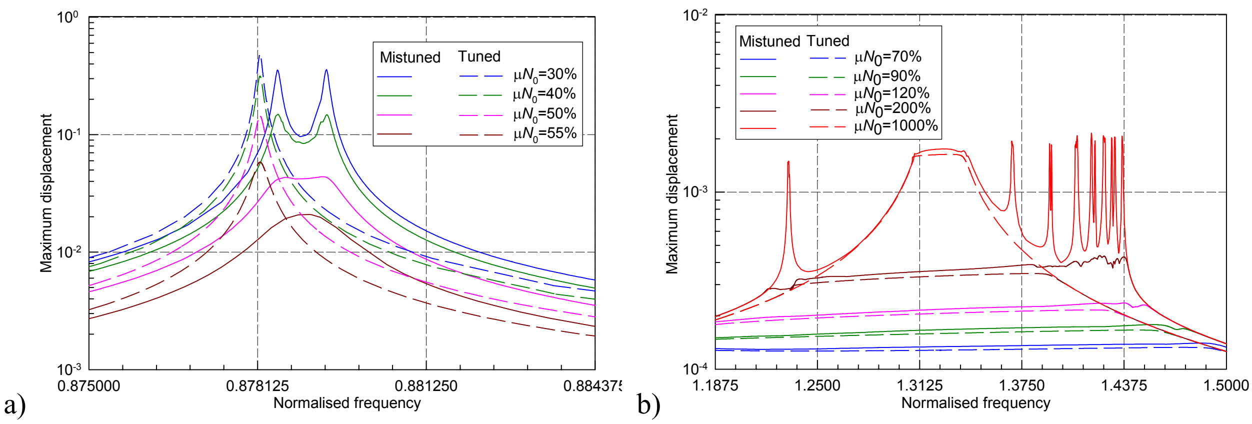


Figure 6: Forced response of the mistuned blisk with underplatform dampers: a) lower, and b) higher values of the limiting friction force

The effect of the choice of the limiting friction force value on the normalised maximum response level, $a_{\Sigma}^{\max}/a_{tuned}^{\max}$ (called also the amplification factor) is shown in Figure 7. One can see that the amplification factor varies over a rather wide range of values: namely, from 0.3 (for $\mu N_0 = 50\%$) to 1.33 (for $\mu N_0 = 1000\%$).

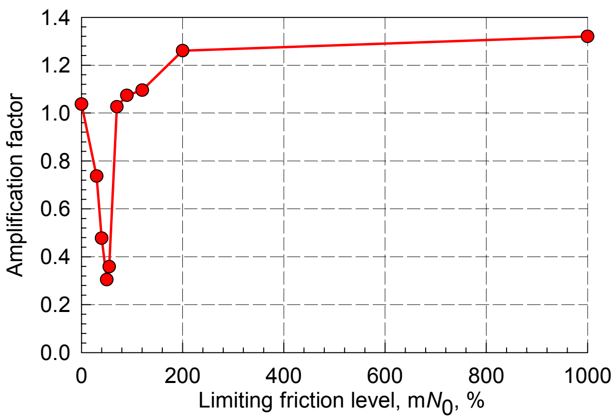


Figure 7: Dependency of the amplification factor on the limiting friction level at UPDs

In Figure 8 forced responses of the blisk mistuned by different ways are compared with the forced response of the tuned blisk. The following mistuned blisks are considered here: (i) a blisk with random blade frequency mistuning (in the range $\pm 1.1\%$) and tuned UPDs; (ii) a blisk with tuned blades and with a scatter in the limiting friction force (in the range $\pm 20\%$). For comparison forced responses of the blisk with tuned blades and tuned UPDs are shown as well. One can see that for $\mu N_0 = 50\%$ mistuning caused by UPD scatter has a very small effect on the forced response in the vicinity of the resonance peak, although for rotation speeds far from the resonance this kind of mistuning increases noticeably the forced response levels. For $\mu N_0 = 200\%$, mistuning caused by UPD scatter has a significant effect on the forced response in a vicinity of the resonance peak and there is no any effect for frequencies far from the resonance. The latter is due to the absence of slip-stick transitions in the dampers at low response levels. It is evident, also, that in the damper-mistuned blisk the transition from fully stuck dampers to slipping dampers happens at a significantly lower level of forced response than in the tuned blisk owing the scatter in the damper parameters and hence to the earlier beginning of slip in the dampers with lower values of the limiting friction force. For UPDs with the friction limiting force $\mu N_0 = 70\%$, which is close to its optimal value and which dampens largely the resonance peak, mistuning caused by UPD scatter has a significant effect over the whole frequency range analysed.

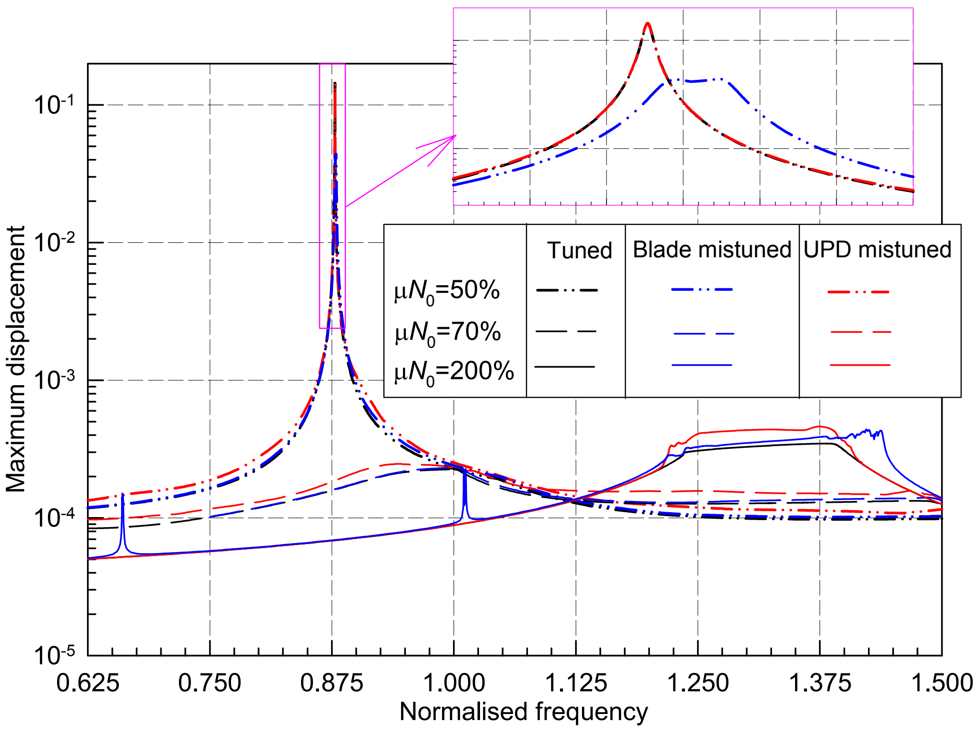


Figure 8: Forced responses of the blisk with mistuned blades and mistuned underplatform dampers

3.2 A high-pressure turbine bladed-disc assembly

Another test case analysed is a high-pressure turbine bladed disc. A finite element model of a quarter of the bladed disc is shown in Figure 9a. The bladed disc has 92 blades and its FE model comprises more than 15 million DOFs. The underplatform dampers are applied to each sector of the assembly at nodes marked by the letter ‘B’ in Figure 9b. The forced response levels are calculated for nodes located at blade tips marked by letter ‘A’ in Figure 9b. To study the nonlinear forced response, the natural frequencies and mass-normalised mode shapes were calculated for all possible nodal diameter numbers from 0 to 46 for the assembly without UPDs. The lower frequency-range natural frequencies calculated for the assembly without UPDs is shown in Figure 9c. Excitation by 6th and 8th engine orders is considered in the frequency ranges indicated in this figure. The modal linear damping loss factor is set in the calculations to 0.003 for all modes included in the modal model. Two major types of the mistuning and their combinations were studied: (i) blade frequency mistuning and (ii) underplatform damper parameter scatter. Blade frequency mistuning was generated in two ways:

(i) A mistuning pattern determined as a result of an optimization search for the worst mistuning pattern providing the highest forced response level developed in [10]. This pattern provides an amplification factor value of 4.9 for the assembly without UPDs (Whitehead's formula in [11] gives a theoretical limit 5.3) when vibrations are excited by 6EO.

(ii) Randomly generated mistuning patterns providing scatter for the first blade-alone frequency in the range $\pm 5\%$ of its nominal value.

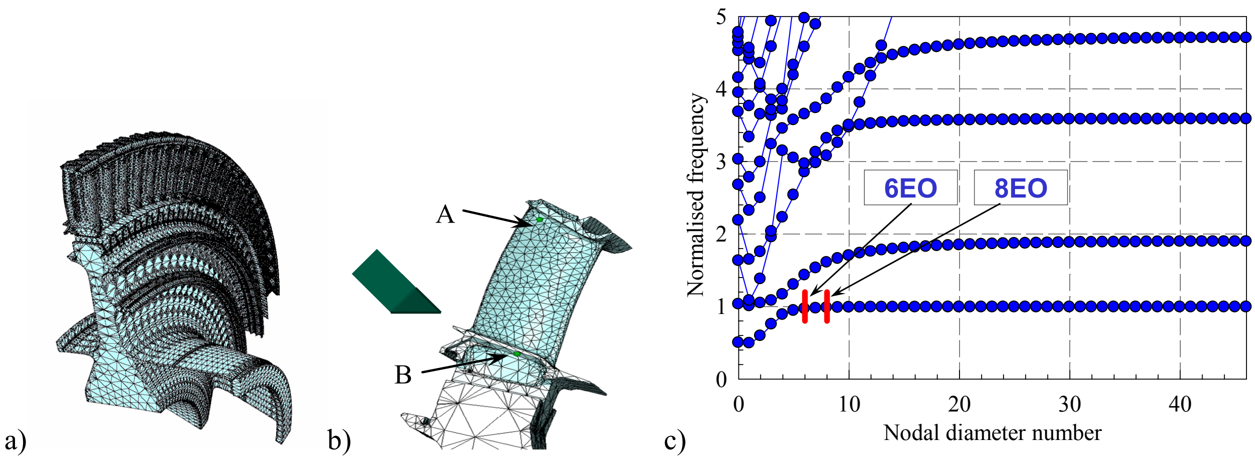


Figure 9: FE model of the turbine high pressure bladed disc

Underplatform damper scatter was set by a random generation of limiting friction forces within a range of $\pm 20\%$ of the nominal value. Stiffness properties were identical for all UPDs. The effects of mistuning on the forced response characteristics were explored for mistuned bladed discs with the following combinations of mistuning sources:

(i) a bladed disc without UPDs and with mistuned blades;
(ii) a bladed disc with mistuned underplatform dampers and tuned blades;
(iii) a bladed disc with tuned underplatform dampers and mistuned blades;
(iv) a bladed disc with mistuned underplatform damper and mistuned blades.

3.2.1 Case of 6EO excitation

Forced responses for 92 blades of the mistuned bladed assembly are shown for a case of 6EO excitation in Figure 10. Four different cases are considered: (i) a bladed disc with mistuned dampers and tuned blades (Figure 10a); (ii) a undamped mistuned bladed disc (Figure 10b); (iii) a blade-mistuned assembly with tuned UPDs (Figure 10c); (iv) a bladed disc with mistuned blades and mistuned UPDs (Figure 10d). The same blade mistuning pattern is used in all cases which provides the high amplification factor value 4.9 for the assembly without UPDs. The envelopes of the forced responses are also plotted in Figure 10 by a thick lines. From Figure 10 one can see that for the case considered, random mistuning of the limiting friction force of the UPDs in the range $\pm 20\%$ produces only a small scatter of the blade amplitudes when blades are tuned. When blades are mistuned for the mistuning pattern analysed, one of the blades has much higher amplitude than all the others over the whole frequency range analysed. This high localization of the maximum response levels is an important effect which should be accounted for in experimental investigations. It is evident that omitting this one blade with the highest response level in measurements reduces the estimate for the maximum response by a significant amount.

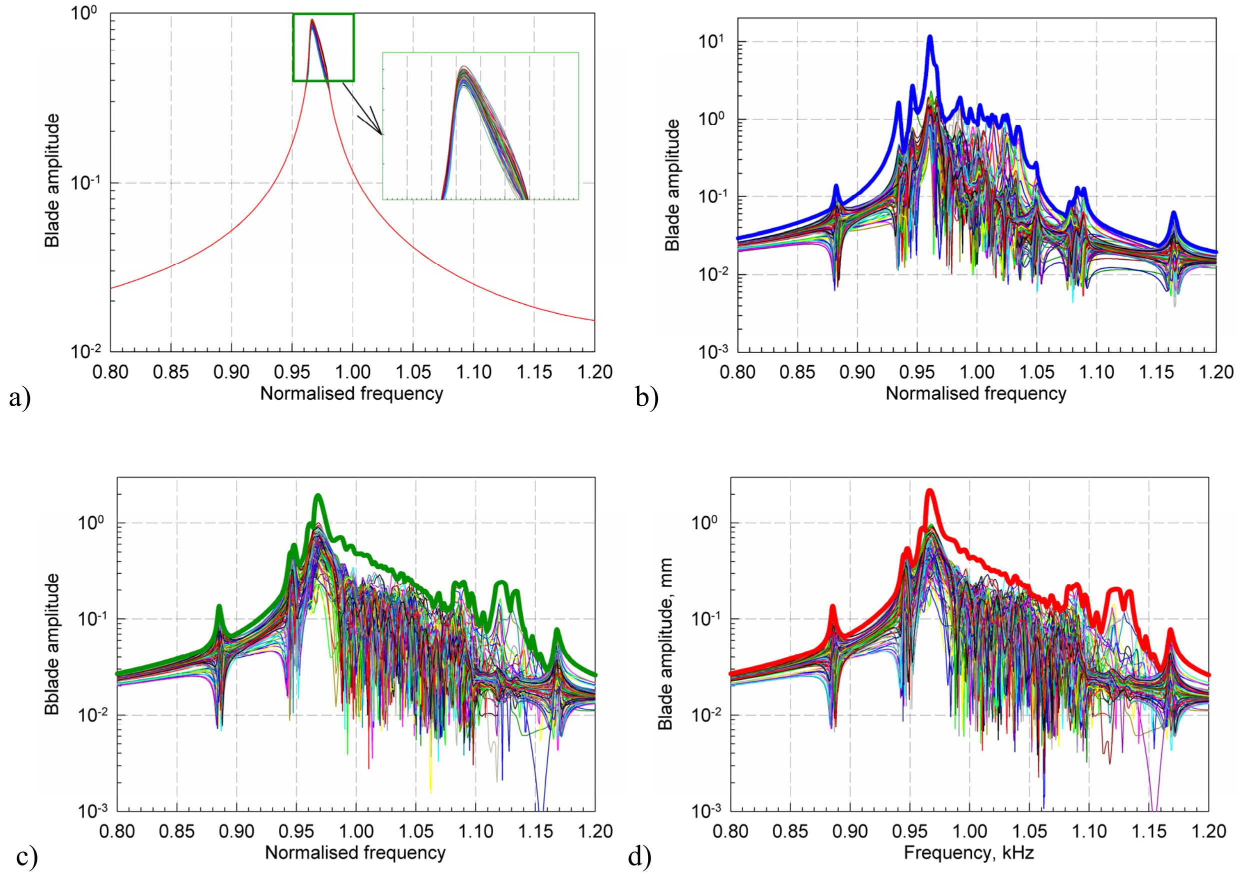
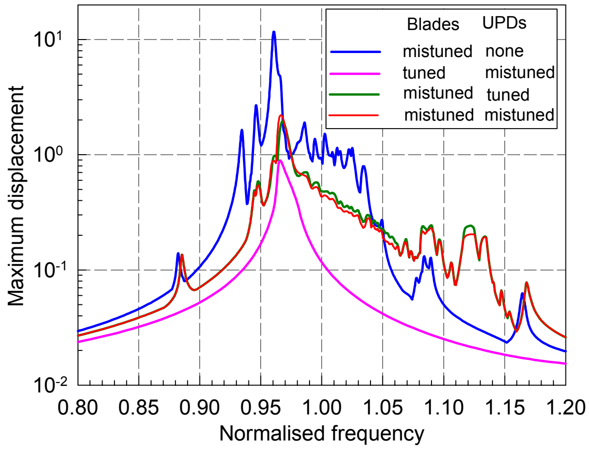


Figure 10: Forced response for each of 92 blades and their envelopes for cases: a) tuned blades and mistuned UPDs; b) mistuned blades without UPDs; c) mistuned blades with tuned UPDs; d) mistuned blades with mistuned UPDs

A comparison of the envelopes of all four differently mistuned assemblies is displayed in Figure 11a. Here, one can see that introduction of the UPDs significantly reduces response levels. The effect of underplatform damper scatter on the forced response level is less significant, for the structure considered, than the effect of the blade frequency mistuning. In contrast to the results obtained for the ADTurbII bladed disc, UPDs cannot fully damp the secondary resonance peaks here, although some of them disappear. The maximum normalised forced response levels found for each blade over the frequency range analysed are plotted in Figure 11b. One can see that the UPDs reduce significantly forced response levels of blades with maximum resonance levels and as result they reduce levels of the localization and scatter of the maximum blade amplitudes.

The amplification factors for bladed discs mistuned by blade frequencies with and without underplatform dampers are compared in Figure 12 for ten different mistuning patterns. The first pattern is the worst mistuning pattern calculated using the optimization search in [10] and all the others are randomly generated mistuning patterns. One can see that for all the calculated mistuned patterns, the amplification factor for the bladed disc with UPDs is significantly smaller that for the structure without UPDs.



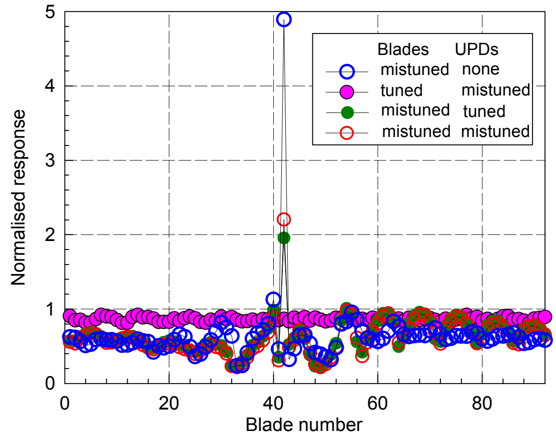


Figure 11: Envelopes of the forced response for the differently mistuned bladed disc (a), and maximum blade amplitudes found over the frequency range analysed (b)

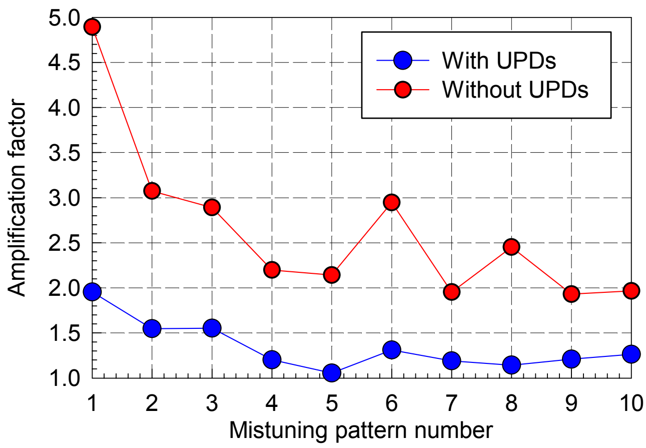


Figure 12: Amplification factors calculated for different mistuning patterns

3.2.2 Case of 8EO excitation

For the case of 8EO excitation, the bladed disc with a mistuning pattern randomly generated for blade frequencies was analysed.

Envelopes of the calculated blade forced responses are plotted in Figure 13a. Similarly to the results obtained for 6EO, one can see here for 8EO that the UPDs significantly reduce forced response levels and that the scatter of UPD's parameters has a secondary significance for this bladed disc. Distributions of the normalised maximum response levels over the blades of the assembly are shown in Figure 13b. It is evident that the scatter of the maximum response levels for mistuned assemblies decreases and the amplification factors are decreased due to introduction of the UPDs.

The amplification factors obtained for these four different combinations of blade and UPD mistuning are summarised in Figure 14. The UPD mistuning causes a noticeable increase in the response levels, although the blade frequency mistuning produces much higher value for the amplification factor. For blade mistuning, additional mistuning due to scatter of the UPDs does not change the amplification factor significantly. As before, the amplification factor for the system with UPDs is significantly smaller that for the system without UPDs, although UPD reduction effects are smaller for 8EO than for 6EO excitation.

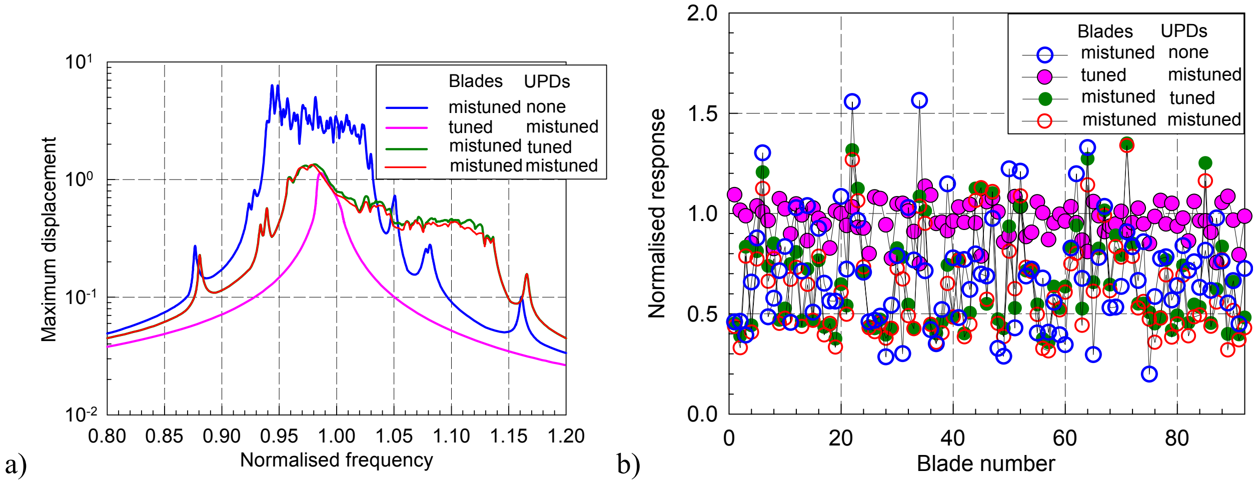


Figure 13: Forced response the turbine high pressure bladed disc excited by 8EO: a) envelopes of the forced response; b) maximum blade amplitudes

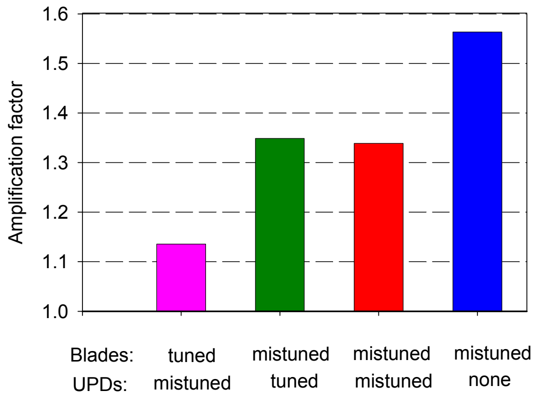


Figure 14: Amplification factors for different kinds of mistuning, 8EO

4.0 CONCLUSIONS

Numerical investigations of nonlinear forced response characteristics for mistuned bladed discs with underplatform dampers have been carried out. Bladed discs of two different designs were analysed: (i) a specially designed test-rig blisk and (ii) an industrial bladed high-pressure turbine disc. Analysis of the results obtained leads to the following practically important observations and conclusions.

- Scatter in UPDs' parameters causes a scatter in blade amplitudes, even when the blades are perfectly tuned.
- The effects of scatter in the limiting friction force/mass in the underplatform dampers on the forced response levels for bladed discs considered are much less significant than those of blade frequency mistuning.
- A mistuned bladed disc with UPDs can have, for certain conditions, smaller response levels than those of its tuned counterpart. For considered cases this effect occurs for a blisk underplatform dampers with small limiting friction force values.
- Underplatform dampers can efficiently suppress secondary resonance peaks (i.e. those resonance peaks which are determined by involvement in the forced response of a mistuned structure of mode shapes with nodal diameter numbers different from the engine-order excitation number) for

a mistuned bladed disc.

- Underplatform dampers can reduce significantly the amplification factors (e.g. from 5 to 2 for some of analysed cases) in addition to the reduction of the forced response levels due to friction damping.
- Underplatform dampers usually reduce the scatter of the maximum blade amplitudes.
- The properties of the underplatform dampers can affect the patterns of distribution of the maximum response levels over blades of the mistuning assembly. This effect is observed for certain conditions even when the underplatform dampers are tuned.

5.0 ACKNOWLEDGEMENT

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SYMPOSIA DISCUSSION – PAPER NO: 38

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Discussor's name: J.P. Lombard

Question: Do you think that the engine manufacturers would have an interest to study or design HP damper devices with voluntary mistuned properties in order to obtain a more reliable dynamic behaviour of bladed discs?

Answer: In my opinion the engine manufacturers should be interested in the assessment of such voluntary mistuning in damper and blade designs. Those effects of forced response levels are expected to be case-dependent.